

MA 692: Financial Maths

Exercise sheet I

Question 1. This question is about discounting, present and future values, and internal rates of return.

(a) Suppose that there is a perpetual cash flow paying the amount

$$x_k = \frac{1}{k!} x^k \quad (1)$$

at year k ($k = 1, 2, 3, \dots$) for some fixed x , and that the annualised rate of return is r (the first payment x_1 is made next year). What is the present value of the cash flow?

(b) Suppose that there is a yearly cash flow $\{x_0, x_1, x_2, \dots, x_n\}$ over n years, where the payment x_0 is made today, and where not all of the x_k 's are positive. What is the equation that defines the internal rate of return \bar{r} ? When the cash flows are given by $\{x_0, x_1, x_2\} = \{-1, \frac{9}{20}, 1\}$, what is the associated internal rate of return \bar{r} ?

(c) Suppose that an amount N is paid now, but also perpetually at the end of each period, and that the per-period interest rate is r . Find the price of this cash flow.

(d) The number of years required to double an investment for a fixed yearly compounded interest rate r must satisfy $(1 + r)^n = 2$. Using second order Taylor series expansion of $\ln(1 + r)$, and for a given number of years n , find an expression for r necessary to double the investment.

(e) (adapted from Luenberger) A young couple has made a non-refundable deposit for one month's rent at a flat, for £1000. The next day, they find another flat which they like, for £800. Should they forfeit their deposit and take the new flat? Assume that the minimum rental term is 6 months, and the interest rate (to calculate the present value) is 12% per annum, compounded monthly. What would be your answer if the minimum term is 1 year?

(f) Suppose that you buy a dilapidated house for £100,000, repair and re-decorate it by spending £15,000 in the process and re-sale it for £135,000. For ease of computation, assume that the repair and redecoration costs were spent at one point in time, exactly 3 months after the purchase, and the house was sold exactly 3 more months afterwards. What is the IRR for this transaction? If the risk-free interest rate to invest for 3 months is 1%, should you invest in this venture? In a project like this, what would be the risks involved?