

MA692: Financial Maths

(Dated: November 22, 2019)

Exercise sheet III

1. Consider a European call option with price 4, maturity 8 months and a strike price 60. If the risk-free rate is 5% per annum with continuous compounding and if the stock price is 58, find the price of a European put option with the same maturity and the same strike price.
2. If the risk-free interest rate corresponding to 12 month maturity is 8% per annum and if the current price of the underlying stock is 100, find the price of a European put option with maturity 12 months and strike price 100 using (i) a one step binomial tree, where the price of the stock is expected to increase or decrease by 20% every 12 month interval and (ii) a two step binomial tree, if the price of the stock is expected to increase or decrease by 10% in every six month interval.
3. Suppose that a fair die is rolled twice. What is the probability that the sum of two throws is greater than or equal to 10?
4. Find the Jensen index and the Sharpe ratio for a stock with a mean return 4%, if the mean return on the market portfolio is 4.5%, the risk-free rate is 1%, the volatility of stock price return is 20% and the volatility of the market portfolio is 25%. The beta of stock is 0.9.

Outline solutions

1. By put-call parity, $C_t + Ke^{-rT} = P_t + S_t$. Hence $P_t = 4 + 60 \exp(-5 * 8/1200) - 58 = 4.03$, up to 2 d.p.
2. For 2 step tree, $T = \frac{1}{2}$, $p = 0.4833$, terminal payoffs (from the top node downwards) are $f_{uu} = 0$, $f_{ud} = 1$, $f_{dd} = 19$; $f_u = 0.4816$, $f_d = 9.6672$, current price $f = 4.9047$ to 4 d.p.
3. The required probability is probability of getting rolls $\{(6 + 4), (6 + 5), (6 + 6), (5 + 5), (5 + 6), (4 + 6)\}$, which is 6 out of 36 outcomes. So the probability is $\frac{1}{6}$.
4. Jensen index is

$$J = \beta(\bar{r}_M - r_f) - (\bar{r} - r_f) = 0.9(3.5) - 3 = 0.15\%$$

and Sharpe ratio is

$$s = \frac{\bar{r} - r_f}{\sigma} = \frac{.03}{0.2} = 0.15.$$

o o o