MA6xx Financial Mathematics assignment

The purpose of this assignment is to demonstrate evidence of understanding of basic principles of mean-variance portfolio optimisation, using real market data. Parts I and II of this assignment can be done with assistance from myself or from TA Pranjal Srivastav. Part III needs to be done on your own and forms 20% of the course assessment.

Part I: preparation

- From `https://in.finance.yahoo.com/quote/%5ENSEI/components?p=%5ENSEI`, download historic stock price data for **any six of the following ten stocks**: Bajaj Auto, Bharati Airtel, Cipla, Grasim, ICICI, L&T (LT), NTPC, Reliance, TCS and Zee. To do this, click on each company's symbol, then click on historical data, then choose time period (**13th July 2017 to 13th July 2019**) and download data, save as xlsx or xls file (not as a csv file).

- Enable Excel add-ins; go to File→ options → Add-Ins and enable Analysis ToolPak and Solver add-in. These should then appear in the Data sub-menu on the toolbar.

- Collect together only the dates (in a single, common column) and the closing price columns of all the stocks you have chosen.

- Now compute the daily returns and the (sample) average return for each of the stocks. Also compute the covariance matrix of the returns. This can be computed, at least partially, by using the data analysis tools. Its commands give you the lower triangular part of the symmetric covariance matrix (covar matrix is always symmetric, *i.e.* - $a_{ij} = a_{ji}$). You can fill up the remaining few entries manually, or use transpose command in EXCEL (ctrl-shift-enter, for any vector or matrix command).

- Choose weights, *e.g.* $w_i = 1/6$, $i = 1, 2, \cdots, 6$, and enter them in a row in your spreadsheet. Try and keep the spreadsheet 'clean', and readable by a third person without help, with proper labelling. Calculate the mean return and the standard deviation of your hypothetical stock portfolio with the chosen weights using EXCEL, as seen on 14th July 2019 and based on two year price history. You want the variance of the portfolio calculated in a cell, on your worksheet. To calculate standard deviation, you need to use *MMULT* and *TRANSPOSE* commands, and use ctrl-shift-enter to execute any matrix commands.

Part II: minimum variance implementation and introductory part of report

There are two sub-parts in this part of your implementation. The first part relates to learning LaTeX and using it to create properly formatted reports.

- If you are not familiar with LaTeX, register on overleaf.com, log-in and upload the zip file called 'latexworkshop.zip' from my Institute webpage, as a project. You can then open project and do the small exercises in the this worksheet to get upto speed with basic LaTeX commands.

- You need to open a new overleaf project and start writing your own report on your numerical experiments with portfolio optimization. Obviously, you can do this on any editor and on offline LaTeX compiler if you wish. Mention the stocks you have chosen, the dates you have chosen, the mean returns and the covariance matrix.

- Separately, use EXCEL's solver to find a minimum variance portfolio for your chosen set of stocks. To do this, have the sum of all your weights calculated in a separate cell (say, H100), which should obviously be 1. Then add a constraint H100=1 (replace 'H100' by whichever cell you have your summation $\sum_i w_i$) in the solver when you do optimisation. Provide information on your minimum variance portfolio in your report.

Part III: Further numerical experiments, submission requirements and marking scheme

- On your spreadsheet, repeat the optimization by adding a target return constraint, $\sum_i w_i \bar{r}_i \geq \alpha$ and minimize variance with this additional constraint for at least five different values of α. Save values of your portfolio weights in all the cases. This gives you five different portfolios, with different mean returns and correspondingly different levels of uncertainty (*i.e.* different standard deviations).

- Find the smallest value of α, if one exists, for which your optimisation problem becomes infeasible, *i.e.* it returns no solution. (2 marks)

- Document your results (all six sets of portfolio weights - one 'pure' minimum variance with no target return constraint and five more with target return constraints) and comment critically on them; *e.g.* is there any stock which has a significant weight in all the portfolios? If so, can you think why? Can you think of a reason for composition of your portfolios and changes in it as you increase α? (7 marks)

- Plot an efficient frontier for your portfolios and insert it in your submission. You can use *excelread* command in matlab if you prefer to plot things in matlab. Otherwise, plot in your favourite software and figure out how to include it in a LaTeX file. (3 marks)

- You need to send two files by email to *paresh.d@iitgn.ac.in*- a PDF file **of a total of 4 pages or less**, sumarizing your numerical experiments and providing appropriate conclusions, and your EXCEL spreadsheet presenting the experiments in a clear enough fashion. Name your flies as 'name_xxxx.yyy' where 'name' is your first name or surname, xxxx are the last four digits of your role number and yyy is either pdf or xlsx (or xls). Also give your name and roll number on the top of the first page of the pdf and in the first worksheet of the EXCEL file. **Fifth page of the pdf, if present, will not be marked**. (General quality of written presentation, including clarity and grammatical correctness, 5 marks, Clarity and correctness of EXCEL sheet, 3 marks).

Some general notes

- **This is NOT financial advice.** 'Top down' techniques such as these are used by fund managers to narrow down the universe of stocks; they will then do some 'bottom

up' analysis of individual stocks (e.g. drivers of company's performance, its leverage etc) before making any investment decision. I don't study or teach the bottom up part as there is little or no maths in it.

- Six stocks do not provide real diversification; 10-12 stocks from different sectors may do so, for an individual. Stocks from different sectors have a lower correlation (and hence give a portfolio with a lower standard deviation), than stocks from the same sector.

- Investment institutions have very large portfolios, *e.g.* one firm I had some work with had a book of over 9000 international stocks.

- Larger number of stocks do not make the optimization any more complex. Any cardinality constraint (*i.e.* a constraint on the **number** of stocks to be included in the portfolio) makes it much harder. If you use 100 stocks instead of 10, the problem would take about $\log 10$ times as long to solve (just believe this). If you put a constraint 'use only 5 out of a given 10 stocks in the portfolio', the problem takes $10C5 \approx 250$ times as long to solve accurately. If you are constructing a portfolio with 50 out of a universe of 1000 stocks, this number is infeasibly large. You need to add cardinality constraints to avoid holding very small number of shares of individual stocks (or, having very small w_i values for most stocks).

- Variance is a poor proxy for risk (although it is good place to start), since it penalizes returns which are too high relative to mean return as well. For those who are interested, better proxies are *conditional value at risk* and *lower partial moments*, both of which penalize only the downside risk and also allow you to generate an efficient frontier etc.

- Portfolios need to be rebalanced periodically, as market conditions change. This needs buying and selling stocks, which involves trading charges, which in turn eats into your return on portfolio.

- Did I mention this is not financial advice? It certainly is not. Do your own research before you invest any *actual* money, or seek professional advice.

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